Math 2FM3, Tutorial 5

Oct 12th, 2015

Increasing Annuities

- The payment series is 1, 2, 3, 4, …, n and each payment at the end of the time section.
- Present value for these payments is

  $(Ia)_{n|i} = v + 2v^2 + 3v^3 + \ldots + nv^n = (\ddot{a}_{n|i} - nv^n)/i$

  when $n \to \infty$, $(Ia)_{\infty|i} = 1/id$ (perpetuity)
- Accumulated value is

  $(Is)_{n|i} = (\ddot{s}_{n|i} - n)/i$

Decreasing Annuity

- The payment series is n, n-1, n-2, …, 2, 1 at the end of each time section.
- Present Value:

  $(Da)_{n|i} = nv + (n-1)v^2 + (n-3)v^3 + \ldots + v^n = (n - a_{n|i})/i$
- Accumulated Value:

  $(Ds)_{n|i} = (n(1+i)^n - s_{n|i})/i = (Da)_{n|i} \cdot (1+i)^n$

Ex 2.3.3

- Jeff and Jason spend X dollars each to purchase annuities. Jeff buys a perpetuity-immediate, which makes annual payments of 30. Jason buys a 10-year annuity-immediate, also with annual payments. The first payment is 53, with each subsequent payment k% larger than the previous year's payment. Both annuities use an effective annual interest rate of k%. Calculate k.

Ex 2.3.5

- A senior executive is offered a buyout package by his company that will pay him a monthly benefit for the next 20 years. Monthly benefits will remain constant within each of the 20 years. At the end of each 12-month period, the monthly benefits will be adjusted upwards to reflect the percentage increase in the CPI. You are given:
- (i) The first monthly benefit is R and will be paid one month from today.
- (ii) The CPI increases 3.2% per year forever.
- At an effective annual interest rate of 6%, the buyout package has a value of 100,000. Calculate R.

Ex 2.3.18

- Joe can purchase one of two annuities:
- Annuity 1: A 10-year decreasing annuity-immediate, with annual payments of 10, 9, 8, …, 1
- Annuity 2: A perpetuity-immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, 3 in year 3, … and 11 in year 11. After year 11, the payments remain constant at 11.
- At an effective annual interest rate of i, the present value of Annuity 2 is twice the present value of Annuity 1. Calculate the value of Annuity 1.